LSM: A DSGE Model for Luxembourg - Appendixes

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1 Appendix A: Derivation of the symmetric equilibrium of LSM

In the following subsections first we specialize the analysis of the production sector and labour market to the case of a CES production function, and then we summarize the equilibrium conditions for the various sectors under the case of a CES production function. The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but now variables are measured in efficiency units.

1.1 The nested CES case

We do not distinguish between tradable and non-tradable goods, but the same production function is assumed in both production processes

$$\begin{aligned} y &= A\left[\alpha k^{\lambda} + (1-\alpha)(\Lambda h)^{\lambda}\right]^{\frac{1}{\lambda}} \\ h &= \left[\varkappa_1 (a_1 h_1)^{\kappa} + \varkappa_2 (a_2 h_2)^{\kappa}\right]^{\frac{1}{\kappa}} \end{aligned} \tag{1}$$

with $\varkappa_2 = 1 - \varkappa_1$. Note that Λ represents a labour-augmenting productivity parameter. We use this nested CES specification since it clearly distinguishes the elasticity of substitution between aggregate labour and capital, and that between the two types of labours. A few additional comments are in order. First, if $\lambda \to 0$ and $\kappa \to 0$, then both CES aggregators collapse to standard Cobb-Douglas forms:

$$\begin{aligned} y &= A k^{\alpha} (\Lambda h)^{1-\alpha} \\ h &= (a_1 h_1)^{\varkappa_1} (a_2 h_2)^{\varkappa_2} . \end{aligned}$$

In this case, it is evident that $\varkappa_j$ represents the share of labor income that accrues to type-j employment. In general, these parameters remain strictly linked to the distribution of income across different types of workers. Second, in (1) only relative labor productivity matters, i.e. a_1/a_2. Finally, we allow for a (purely external) effect of the stock of public infrastructure ($INFR_t$) on the Total Factor Productivity, A. In particular, we model A as:

$$A = (INFR_t)^{\varpi} \cdot EXOG,$$

where $0 < \varpi < 1$, $EXOG$ represents exogenous technical progress growing at a constant rate γ. Note also that:

$$
\begin{aligned}
\frac{\partial y}{\partial h} &= (\Lambda A)^{\lambda}(1-\alpha)\left(\frac{h}{y}\right)^{\lambda-1}, \\
\frac{\partial h}{\partial h_z} &= \varkappa_z a_z^{\kappa}\left(\frac{h_z}{h}\right)^{\kappa-1}, \\
\frac{\partial^2 y}{\partial h^2} &= (\lambda-1)\frac{\partial y}{\partial h}\left[1-\frac{(1-\alpha)h^{\lambda}}{\alpha(\Lambda k)^{\lambda}+(1-\alpha)h^{\lambda}}\right]\frac{1}{h}, \\
\frac{\partial^2 h}{\partial h_z^2} &= (\kappa-1)\frac{\partial h}{\partial h_z}\left[1-\varkappa_z\left(\frac{a_z h_z}{h}\right)^{\kappa}\right]\frac{1}{h_z}.
\end{aligned}
$$

It follows that the first order conditions of the firm can be written as:

$$
\begin{aligned}
\frac{p}{\mu}(\Lambda A)^{\lambda}(1-\alpha)\left(\frac{h}{y}\right)^{\lambda-1}\varkappa_z a_z^{\kappa}\left(\frac{h_z}{h}\right)^{\kappa-1} &= (1+\tilde{\tau}_L)\,w_z \\
\frac{p}{\mu}A^{\lambda}\alpha\left(\frac{k}{y}\right)^{\lambda-1} &= r
\end{aligned}
$$

Then we have:

$$
\begin{aligned}
\frac{rk}{py} &= \frac{1}{\mu}\alpha\left(A\frac{k}{y}\right)^{\lambda}, \\
\frac{(1+\tilde{\tau}_L)\sum_{j=1}^{2} w_j h_j}{py} &= \frac{1}{\mu}(1-\alpha)\left(A\Lambda\frac{h}{y}\right)^{\lambda}, \\
\frac{(1+\tilde{\tau}_L)\,w_z h_z}{py} &= \frac{1}{\mu}(1-\alpha)\left(A\Lambda\frac{h}{y}\right)^{\lambda}\varkappa_z\left(\frac{a_z h_z}{h}\right)^{\kappa} = \\
&\quad \frac{(1+\tilde{\tau}_L)\sum_{j=1}^{2} w_j h_j}{py}\varkappa_z\left(\frac{a_z h_z}{h}\right)^{\kappa},
\end{aligned}
$$

and for the labour market:

$$
\epsilon_z = \left\{\left[\frac{1-\lambda\mu}{\mu}(1-\alpha)\left(A\Lambda\frac{h}{y}\right)^{\lambda}+\lambda-\kappa\right]\varkappa_z\left(\frac{a_z h_z}{h}\right)^{\kappa}+\kappa-1\right\}^{-1}, \quad z=1,2.
$$

1.2 Households

The key equations for the Household sector of LSM are:

$$
\begin{aligned}
C_{t+1} &= \frac{\mathcal{E}_{t+1}}{\gamma}\left(C_t - \frac{\eta-\varphi}{\eta}\frac{A_t}{\zeta_t - \mathcal{Z}_t}\right) \\
D_t &= \xi_t C_t \\
A_t &= R_t \frac{A_{t-1}}{\gamma\eta} + \mathrm{W}_t - \mathcal{Z}_t C_t \\
\mathrm{W}_t &= \left(1-\tau_L\right)\left[w_{1,t}H_{1,t} + \bar{w}_{1,t}\left(1-H_{1,t}\right)\right] + \left(1-\tau_K\right)\Pi_t + \varrho_1 \bar{G}_t \\
\zeta_t &= \mathcal{Z}_t + \mathcal{E}_{t+1}\frac{\varphi}{R_{t+1}}\zeta_{t+1} \\
\xi_t &= \left\{\frac{\phi}{1-\phi}\left[\varkappa_t^d - \frac{\varkappa_{t+1}^d\left(1-\delta_D\right)}{R_{t+1}\frac{p_t}{p_{t+1}}}\right]\right\}^{\frac{1}{\upsilon-1}} \\
\mathcal{E}_t &= \left\{\left[\frac{\phi + \left(1-\phi\right)\xi_t^{\upsilon}}{\phi + \left(1-\phi\right)\xi_{t-1}^{\upsilon}}\right]^{\frac{1-\upsilon-\sigma}{\upsilon}}\beta R_t \frac{p_{t-1}}{p_t}\right\}^{\frac{1}{\sigma}} \\
\mathcal{Z}_t &= \left(1+\tau_C\right)p_t\left[1 + \varkappa_t^d\left(\xi_t - \frac{1-\delta_D}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_t}\right)\right]
\end{aligned}
$$

1.3 Asset Stock

The key equations for the Asset Stock sector of LSM are:

$$
\begin{aligned}
F_t &= A_t - B_t - p_t \nu_t K_t \\
K_t &= \left[1 - \delta_K + \frac{\Xi_1}{\varsigma}\left(\gamma\eta\frac{I_t}{K_{t-1}}\right)^{\varsigma} + \Xi_2\right]\frac{K_{t-1}}{\gamma\eta} \\
\nu_t &= \frac{1}{\Xi_1}\left(\gamma\eta\frac{I_t}{K_{t-1}}\right)^{1-\varsigma} \\
p_t\nu_t &= \frac{\left(1-\tau_K\right)r_{t+1} + p_{t+1}\left(\tau_K\delta_K - \frac{\gamma\eta I_{t+1}}{K_t}\right)}{R_{t+1}} \\
&\quad + \frac{p_{t+1}\nu_{t+1}\left[1 - \delta_K + \frac{\Xi_1}{\varsigma}\left(\frac{\gamma\eta I_{t+1}}{k_t}\right)^{\varsigma} + \Xi_2\right]}{R_{t+1}}
\end{aligned}
$$

1.4 Final good sector

$$
\begin{aligned}
Y &= \mathcal{N}^{\rho-\mu}\left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}}\right)^{\mu} \\
y_j &= \left(\frac{p_j}{p}\right)^{\frac{\mu}{1-\mu}} Y\mathcal{N}^{\frac{\rho-\mu}{\mu-1}} \\
p &= \mathcal{N}^{-(\rho-\mu)}\left(\sum_{j=1}^{\mathcal{N}} p_j^{\frac{1}{1-\mu}}\right)^{1-\mu}
\end{aligned}
$$

1.5 Intermediate goods sector

1.5.1 Non-tradable goods

The key equations for the non-tradable goods sector and associated labour market are:

$$
\begin{aligned}
y^{NT} &= A\left[\alpha(k^{NT})^{\lambda}+(1-\alpha)\left(\Lambda h^{NT}\right)^{\lambda}\right]^{\frac{1}{\lambda}} \\
h^{NT} &= \left[\varkappa_1\left(a_1 h_1^{NT}\right)^{\kappa}+\varkappa_2\left(a_2 h_2^{NT}\right)^{\kappa}\right]^{\frac{1}{\kappa}} \\
\mu\left(1+\tilde{\tau}_L\right) w_z^{NT} &= p^{NT}(1-\alpha)(A\Lambda)^{\lambda}\varkappa_z a_z^{\kappa}\left(\frac{y^{NT}}{h^{NT}}\right)^{1-\lambda}\left(\frac{h^{NT}}{h_z^{NT}}\right)^{1-\kappa} \\
\mu r &= p^{NT}\alpha A^{\lambda}\left(\frac{y^{NT}}{k^{NT}}\right)^{1-\lambda} \\
p^{NT} &= \mathcal{N}^{\frac{\rho-\mu}{\mu}}\left(\frac{y^{NT}}{Y}\right)^{\frac{1-\mu}{\mu}} p \\
\epsilon_z &= \left\{\begin{array}{c}\left[\frac{1-\lambda\mu}{\mu}(1-\alpha)\left(A\Lambda\frac{h^{NT}}{y^{NT}}\right)^{\lambda}+\lambda-\kappa\right]\times \\ \varkappa_z\left(\frac{a_z h_z^{NT}}{h^{NT}}\right)^{\kappa}+\kappa-1\end{array}\right\}^{-1} \\
\theta_z\left(1+\frac{w_z^{NT}-\overline{w}_z}{w_z^{NT}}\epsilon_z^{NT}\right)\frac{\tilde{\pi}^{NT}}{h_z^{NT}} &= \left(1+\tilde{\tau}_L\right)\left(1-\theta_z\right)\left(w_z^{NT}-\overline{w}_z\right) \\
\pi^{NT} &\equiv \left(1-\frac{1}{\mu}\right)p^{NT}y^{NT}-\psi \\
\tilde{\pi}^{NT} &= p^{NT}y^{NT}-\left(1+\tilde{\tau}_L\right)\sum_{s=1}^{2} w_s^{NT}h_s^{NT}
\end{aligned}
$$

1.5.2 Tradable goods

The key equations for the tradable goods sector and associated labour market are:

$$
\begin{aligned}
y^T &= A\left[\alpha(k^T)^\lambda + (1-\alpha)\left(\Lambda h^T\right)^\lambda\right]^{\frac{1}{\lambda}} \\
h^T &= \left[\varkappa_1\left(a_1 h_1^T\right)^\kappa + \varkappa_2\left(a_2 h_2^T\right)^\kappa\right]^{\frac{1}{\kappa}} \\
\mu\left(1+\tilde{\tau}_L\right) w_z^T &= p^T(1-\alpha)(A\Lambda)^\lambda \varkappa_z a_z^\kappa \left(\frac{y^T}{h^T}\right)^{1-\lambda}\left(\frac{h^T}{h_z^T}\right)^{1-\kappa} \\
\mu r &= p^T \alpha A^\lambda \left(\frac{y^T}{k^T}\right)^{1-\lambda} \\
p^T &= p^H = p^F \\
p^H &= \mathcal{N}^{\frac{\rho-\mu}{\mu}}\left(\frac{s^H y^T}{Y}\right)^{\frac{1-\mu}{\mu}} p \\
p^F &= \left(1-t^F\right)\left(\mathcal{N}^*\right)^{\frac{\rho-\mu}{\mu}}\left(\frac{s^F y^T}{Y^*}\right)^{\frac{1-\mu}{\mu}} p^* \\
s^F &= 1-s^H \\
\epsilon_z &= \left\{\begin{array}{c}\left[\frac{1-\lambda\mu}{\mu}(1-\alpha)\left(A\Lambda\frac{h^T}{y^T}\right)^\lambda + \lambda-\kappa\right]\times \\ \varkappa_z\left(\frac{a_z h_z^T}{h^T}\right)^\kappa + \kappa - 1\end{array}\right\}^{-1} \\
\theta_z\left(1+\frac{w_z^T-\overline{w}_z}{w_z^T}\epsilon_z^T\right)\frac{\tilde{\pi}^T}{h_z^T} &= \left(1+\tilde{\tau}_L\right)\left(1-\theta_z\right)\left(w_z^T-\overline{w}_z\right) \\
\tilde{\pi}^T &= p^T y^T - \left(1+\tilde{\tau}_L\right)\sum_{s=1}^{2} w_s^T h_s^T \\
\pi^T &\equiv \left(1-\frac{1}{\mu}\right)p^T y^T - \psi
\end{aligned}
$$

1.5.3 Importers

For the imported good sector we have:

$$
\begin{aligned}
p^M &= \mu\left(1+t^M\right)p_M^* \\
y^M &= \left(\frac{\mu\left(1+t^M\right)p_M^*}{p\mathcal{N}^{\frac{\rho-\mu}{\mu}}}\right)^{\frac{\mu}{1-\mu}} Y \\
\pi^M &\equiv (\mu-1)\left(1+t^M\right)p_M^* y^M - \psi_j
\end{aligned}
$$

1.6 Aggregation

The aggregate variables are given by

$$
\begin{aligned}
Y &= \mathcal{N}^{\rho-\mu}\left[\Theta \mathrm{N}\left(y^{NT}\right)^{\frac{1}{\mu}}+(1-\Theta)\,\mathrm{N}\left(y^{H}\right)^{\frac{1}{\mu}}+(1-\Theta^{*})\,\mathrm{N}^{*}\left(y^{M}\right)^{\frac{1}{\mu}}\right]^{\mu} \\
P &= \mathcal{N}^{\mu-\rho}\left[\begin{array}{c}\Theta \mathrm{N}\left(p^{NT}\right)^{\frac{1}{1-\mu}}+(1-\Theta)\,\mathrm{N}\left(p^{H}\right)^{\frac{1}{1-\mu}} \\ +(1-\Theta^{*})\,\mathrm{N}^{*}\left(p^{M}\right)^{\frac{1}{1-\mu}}\end{array}\right]^{1-\mu} \\
H_z &= \left[\Theta h_z^{NT}+(1-\Theta)\,h_z^{T}\right]\mathrm{N} \\
w_z &= \frac{w_z^{NT}\Theta h_z^{NT}+w_z^{T}\,(1-\Theta)\,h_z^{T}}{\Theta h_z^{NT}+(1-\Theta)\,h_z^{T}} \\
K &= \left[\Theta k^{NT}+(1-\Theta)\,k^{T}\right]\mathrm{N} \\
\Pi &= \left[\Theta \pi^{NT}+(1-\Theta)\,\pi^{T}\right]\mathrm{N}+(1-\Theta^{*})\,\mathrm{N}^{*}\pi^{M}
\end{aligned}
$$

1.7 Numeraire

We set the numeraire as the price of the non-traded goods:

$$p^{NT}=1$$

1.8 Government

The key equations for the Government sector are:

$$
\begin{aligned}
B_t &= R_t\frac{B_{t-1}}{\gamma\eta}+G_t-T_t \\
T_t &= \tau_K\left[i_tF_{t-1}+(r_t-\delta_Kp_t)\,K_{t-1}+\Pi_t\right]+ \\
&\quad (\tau_L+\tilde{\tau}_L)\,(w_{1,t}H_{1,t}+w_{2,t}H_{2,t})+\tau_L\bar{w}_{1,t}\,(1-H_{1,t})+ \\
&\quad \tau_Cp_t\left[1+\varkappa_t^d\left(\xi_t-\frac{1-\delta_D}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_t}\right)\right]C_t+ \\
&\quad \tau_M\,(1-\Theta^{*})\,\mathrm{N}^{*}p_M^{*}y^M \\
G_t &= \bar{w}_{1,t}\,(1-H_{1,t})+TR_t^F\,(\tau_L+\tilde{\tau}_L)\,w_{2,t}H_{2,t}+\bar{G}_t \\
TR_t &= \varrho_1\bar{G}_t \\
GCON_t &= \varrho_2\bar{G}_t \\
INFR\_INV_t &= (1-\varrho_1-\varrho_2)\,\bar{G}_t \\
INFR_t &= (1-\delta_{INFR})\,INFR_{t-1}+INFR\_INV_t \\
\bar{G}_t &= \vartheta\bar{G}_{t-1}+(1-\vartheta)\,d^{LR}\left[\begin{array}{c}T_t-\bar{w}_{1,t}\,(1-H_{1,t}) \\ -TR_t^F\,(\tau_L+\tilde{\tau}_L)\,w_{2,t}H_{2,t}\end{array}\right] \\
\bar{w}_{1,t} &= rep_1NETINC_t \\
\bar{w}_{2,t} &= rep_2NETINC_t
\end{aligned}
$$

where:

$$
\begin{aligned}
NETINC_t &= (1-\tau_L)\left[w_{1,t}H_{1,t} + \bar{w}_{1,t}(1-H_{1,t})\right] + \\
&\quad \left[(1-\tau_K)r_t + \tau_K\delta_K p_t\right]K_{t-1} + (1-\tau_K)\Pi_t
\end{aligned}
$$

The last two equations determine unemployment benefits as a function of net income.

1.9 Exogenous variables

The following variables are treated as exogenous in LSM:

$$
\begin{aligned}
R_t &\equiv 1 + (1-\tau_K)\, i_t \\
i_t &= \bar{\imath} + \xi_i \left[\exp\left(\bar{f} - \frac{F_t}{GDP_t}\right) - 1\right] + \varepsilon_{it} \\
A &= A_0 \left(INFR_t\right)^{\varpi}
\end{aligned}
$$

2 Appendix B: LSM parameters and their calibrated value

As discussed in the main text, we can divide the LSM parameters into three groups according to the way we set their values. Here we provide additional details for the parameters of each group.

2.1 Parameter values based on other papers in the literature or theoretical considerations

- β : the subjective discount factor. We set this parameter to 0.995.
- υ : the parameter related to the elasticity of substitution between consumption and dwellings in the utility function. We set the parameter in order to reproduce an elasticity of substitution equal to 1.5.
- σ : this parameter equals $1/\sigma^c$, where σ^c is the elasticity of intertemporal substitution. We assume logarithmic preferences, i.e. we set the parameter equal to unity.
- δ_K : the depreciation rate of physical capital. Following Backus, Henriksen, and Storesletten (2008), we choose a value of 8.5%.
- δ_D : the depreciation rate of the stock of dwellings. Again, following Backus, Henriksen, and Storesletten(2008), we set the parameter equal to 1.5%.
- δ_{INFR} : the depreciation rate of the stock of public infrastructure. The same reference as before suggests a value of 4.15%.
- α : the relative weight of physical capital in the CES production function. This parameter is strictly related to the capital share in output (actually, under a Cobb-Douglas specification, the two coincide). We set the parameter equal to 0.36, a standard value. The implied capital share in production under the benchmark parameterization lies around 25%.
- ξ_i the elasticity of the international interest rate with respect to the national debt/GDP ratio. Following Schmitt-Grohe and Uribe (2004), we set the parameter equal to 0.000742.
- TR_t^F : the percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers. We choose a value equal to 0.6.
- ϑ : the persistence of core government expenditure. We choose a value equal to 0.9.

- ς : the elasticity of the adjustment cost with respect to the investment-capital ratio. Following Boldrin, Christiano, and Fisher (2001), we set the parameter equal to $1 - 1/0.23$.
- Θ : the share of non-traded domestic varieties. We set the parameter equal to 0.5.
- N : the number of available domestic differentiated intermediate goods. We set the value equal to 2.
- Θ^* : the share of traded foreign varieties (the share of importable varieties into Luxembourg). We choose a value equal to 0.5 for the sake of symmetry.
- N^* : the number of available foreign differentiated intermediate goods. We choose a value equal to 2, again for the sake of symmetry.
- ρ : the parameter capturing the increasing returns to variety. We assume no returns to variety in the benchmark parametrization, and set the parameter equal to 1.
- μ : the parameter related to the elasticity of substitution among intermediate goods. We set the parameter to obtain an elasticity equal to 1.5.
- ψ_j : the fixed cost to enter the market of intermediate good j. We choose a small value equal to 0.00001.
- θ_z : the relative bargaining power of the union for type z workers. We choose a value equal to 0.5.
- P^* : the foreign aggregate price level. Normalized to unity.
- Λ : labour-augmenting productivity parameter. We normalize it to unity.
- a_1 : the parameter augmenting type-1 labour in the labour CES aggregator. It is normalized to unity.
- a_2 : the parameter augmenting type-2 labour in the labour CES aggregator. It is normalized to unity.
- κ : the parameter related to the elasticity of substitution between the two labour types in the CES labour aggregator. We set the value of the parameter in order to obtain an elasticity equal to 1.5.
- ϖ : the parameter related to the elasticity of TFP with respect to public infrastructure. We choose a value equal to 0.01.

2.2 Parameter values inferred from direct evidence on the value of the parameter:

- φ : the individual survival rate, i.e. at the individual level, one minus the probability of dying at the end of the current period. Average life expectancy at birth in Luxembourg was 79.18 years in 2008 (CIA factbook): the survival rate that reproduces this outcome is 0.987.

- $\bar{f}$: The steady-state net foreign position relative to GDP. The average value of net foreign position was 95% and 75% of GDP at the end of 2007 and 2008, respectively (according to the bulletin of the Luxembourg Central Bank). Thus, we set the parameter to 0.85.

- τ_C : the tax rate on consumption (both durables and non-durables). We choose a value of $25,1\%$, taken from *Taxation trends in the EU*, European Commission, 2008. Note that the tax base for consumption tax includes non-durables consumption expenditure and the investment in durables.

- τ_L : the tax rate on labour related income, paid by the employee. We follow again *Taxation trends in the EU*, 2008, and set the value to 20.1%. The figure has been obtained this way: the total average effective tax rate on labour equals $29,6\%$, but only $67,9\%$ of this amount is paid by the employee. Hence, the average effective tax rate on labour paid by the employee becomes 20.1%.

- $\tilde{\tau}_L$: the social contribution rate on labour related income, paid by the employer. Given the previous result, we set the parameter to 9.5%.

- τ_K : the tax rate on profits and capital income. Because of data availability problems, the source *Taxation trends in the EU*, 2008, does not report an estimate of the average effective tax rate on capital. We take the average effective tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.

- η : the population growth rate. We set the parameter equal to 1.012, since the current population growth rate in Luxembourg is 1.2% (data from CIA factbook, year 2008).

- γ : the rate of exogenous long-run technological progress. We set this parameter equal to 0.6%, which is the average TFP growth rate in Luxembourg over the 1995-2009 period, as reported in the Annual Report of the Luxembourg Central Bank (2006, p. 54).

- t^M : the tariff on imported goods. The Overall Trade Restrictiveness Index in 2006 for the European Union equals 6.6%, as computed by the World Bank. This index is the ad-valorem equivalent of all tariff and non-tariff barriers that a country imposes on foreign imports. However, in 2007 94.5% of all imported goods originated from countries within the EEA and no tariff was applied in Luxembourg. Thus, the effective tariff on imported goods is 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares.

- t^F : the tariff on exported goods. As before, in 2007 88.2% of all exported goods were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate equal to 9%, which is the MA-OTRI in 2006 for the European Union. This is the ad-valorem equivalent of all tariff and non-tariff barriers that a country faces as an exporter. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.

- λ : the parameter related to the elasticity of substitution between capital and labour in the CES production function. Guarda (1997) estimates the elasticity to be 1.012 in the tradables sector. We set the value of the parameter in order to obtain the elasticity equal to 1.012.

- χ_1 : the share of type-1 labour in the labour CES aggregator. We choose a value equal to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.

2.3 Parameter values calibrated so the model matches observed ratios in the data:

- ϕ : the relative weight of durables and non durables consumption in the utility function. We calibrate the parameter in order to reproduce the share of durable goods consumption expenditure from the final consumption expenditure of households equal to 0.116 (average annual share between 1995-2008). The implied value of ϕ is 0.898.

- ϱ_1 : the share of transfers to resident households in core (government) expenditure. We set the parameter equal to 43.154%, in order to make the model replicate the share of government transfers in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).

- ϱ_2 : the share of public investment in infrastructures in core (government) expenditure. We set the parameter equal to 11.369%, in order to make the model replicate the share of government investment in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).

- $\bar{\imath}$ the constant and exogenous long-run interest rate if the country settles down to a net foreign position equal to its steady-state value (interest rate risk premium is zero). We calibrate its value to match the observed net foreign position at 85% of GDP in Luxembourg (represented by $\bar{f}$). The implied value equals 2.035%.

- d^{LR} : the parameter related to the long-run debt/GDP ratio. We calibrate the parameter in order to reproduce the observed debt/GDP ratio of Luxembourg at 0.069. The implied value for the parameter is 1.0009212.

- Y^* and p_M^*: the foreign real output level and the price of imported goods. We calibrate them in order to reproduce a net exports to GDP ratio equal to about 0.35. The implied values are, respectively, 5.016 and 0.524.

- $REP1$: replacement ratio of unemployment benefit for domestic workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 21.945% in order to replicate a 5% unemployment rate of type-1 workers.

- $REP2$: replacement ratio of unemployment benefit for foreign workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 15.987% in order to replicate the ratio of type-1 to type-2 workers equal to 1.4238.